Many real-world systems arising in science, engineering, and finance exhibit behaviour across multiple scales, where small changes can produce significant effects. A central mathematical framework for understanding such phenomena is *singular perturbation theory*, which studies problems in which a small parameter leads to sharp transitions or rapid variations in the solution. These localized changes, often referred to as boundary or interior layers, present significant challenges for standard numerical methods.

A simple way to understand this is through a real-life analogy. Imagine driving on a highway where most of the road is smooth, but suddenly you encounter a sharp speed breaker or a rough patch. Although the irregularity is confined to a small region, it significantly affects the overall motion of the vehicle. Similarly, in singular perturbation problems, small regions with rapid changes can strongly influence the overall behaviour of the system.

Dr. Satpal Singh's research focuses on developing robust and accurate computational techniques to address these challenges. By employing advanced discretization strategies, such as exponentially graded meshes, the numerical methods are specifically designed to capture sharp gradients efficiently. These approaches ensure that computational effort is concentrated in regions where the solution changes rapidly, leading to improved accuracy without excessive cost.

The work is grounded in mathematical modelling, with particular relevance to computational finance. Financial systems, especially in areas such as option pricing and risk analysis, often display behaviour analogous to singular perturbation problems—where sudden changes occur under certain conditions. By leveraging parameter–uniform numerical methods, the proposed framework provides stable and reliable approximations across varying regimes.

To enhance the approximation quality, spline-based techniques, including quadratic B-splines, are incorporated to achieve smooth and accurate representations of the solution. These methods are supported by rigorous theoretical analysis, including the derivation of derivative bounds and proofs of second-order parameter–uniform convergence, ensuring both accuracy and reliability.

An emerging component of this research is the integration of Physics-Informed Neural Networks (PINNs), which combine machine learning with governing mathematical principles. By embedding differential equations into the learning process, PINNs offer a powerful tool for handling complex and high-dimensional problems. When guided by insights from singular perturbation theory, these methods become particularly effective in capturing sharp solution features.

Overall, this work presents a unified approach that combines classical analysis, advanced numerical methods, and modern computational techniques. It aims to transform complex mathematical models into practical tools for understanding and predicting real-world systems with greater accuracy and confidence.